



# ICPC SOUTH PACIFIC REGIONALS

NOVEMBER 23, 2019

---

## Contest Problems

---

A: Amazing Sushi  
B: Bubble Bucket Sort  
C: Cartography  
D: Divisionals Spelling  
E: Entering Rectangles  
F: Fabulous Photos  
G: Grid City  
H: Heroic Heist  
I : Iguana Instructions  
J : Jumping Junipers  
K: Karaoke Meetup

This contest contains eleven problems over 24 pages. Good luck.

For problems that state “*Your answer should have an absolute or relative error of less than  $10^{-9}$* ”, your answer,  $x$ , will be compared to the correct answer,  $y$ . If  $|x - y| < 10^{-9}$  or  $\frac{|x - y|}{|y|} < 10^{-9}$ , then your answer will be considered correct.

---

### Definition 1

For problems that ask for a result modulo  $m$ :

If the correct answer to the problem is the integer  $b$ , then you should display the unique value  $a$  such that:

- $0 \leq a < m$   
and
- $(a - b)$  is a multiple of  $m$ .

---

### Definition 2

A string  $s_1 s_2 \dots s_n$  is lexicographically smaller than  $t_1 t_2 \dots t_\ell$  if

- there exists  $k \leq \min(n, \ell)$  such that  $s_i = t_i$  for all  $1 \leq i < k$  and  $s_k < t_k$   
or
- $s_i = t_i$  for all  $1 \leq i \leq \min(n, \ell)$  and  $n < \ell$ .

---

### Definition 3

- Uppercase letters are the uppercase English letters ( $A, B, \dots, Z$ ).
- Lowercase letters are the lowercase English letters ( $a, b, \dots, z$ ).

---

### Definition 4

Unless otherwise specified, the distance between two points  $(x_0, y_0)$  and  $(x_1, y_1)$  is defined as its Euclidean distance:

$$\sqrt{(x_0 - x_1)^2 + (y_0 - y_1)^2}.$$

---

### Definition 5

A positive number is a number that is strictly greater than 0, while a negative number is strictly less than 0. This means that zero is neither negative nor positive. A number is nonpositive if it is not positive and a number is nonnegative if it is not negative.

# Problem A

## Amazing Sushi

Time limit: 1 second

Mary and Marty were playing with their Marvelous Marble Machine and have become hungry. So, they decided to order some sushi. There are several types of sushi. The sushi platter comes with pieces of various types (possibly many of each type).

Both Mary and Marty know a range of the total number of pieces of sushi they can eat. They would like to know if there is a way to distribute their sushi such that neither of them eats too little or too much sushi and no sushi goes uneaten. To be fair, Mary and Marty each want to eat half of the pieces of each type. If there is an odd number of pieces for a given type, either one of them can eat the extra piece.



Does there exist a way for Mary and Marty to properly distribute their sushi?

### Input

The first line contains a single integer  $n$  ( $1 \leq n \leq 100$ ), which is the number of types of sushi.

The second line describes the number of pieces of sushi Mary can eat. This line contains two integers,  $x_1$  ( $0 \leq x_1 \leq 100\,000$ ) and  $y_1$  ( $x_1 \leq y_1 \leq 100\,000$ ). Mary must eat at least  $x_1$  pieces and at most  $y_1$  pieces.

The third line describes the number of pieces of sushi Marty can eat. This line contains two integers,  $x_2$  ( $0 \leq x_2 \leq 100\,000$ ) and  $y_2$  ( $x_2 \leq y_2 \leq 100\,000$ ). Marty must eat at least  $x_2$  pieces and at most  $y_2$  pieces.

The next  $n$  lines describe the  $n$  different types of sushi in Mary and Marty's platter. Each line contains a single integer  $m$  ( $1 \leq m \leq 1\,000$ ), which is the number of pieces of sushi of this type.

### Output

If there exists a way for Mary and Marty to properly distribute their sushi, display Yes. Otherwise, display No.

#### Sample Input 1

```
3
1 10
7 20
5
3
1
```

#### Sample Output 1

No

#### Sample Input 2

```
3
1 10
3 20
5
3
14
```

#### Sample Output 2

Yes

**Sample Input 3**

```
3
1 10
3 20
5
3
16
```

**Sample Output 3**

No



## Problem B

### Bubble Bucket Sort

Time limit: 1 second

Belinda loves blowing bubbles! After a long day of blowing bubbles, Belinda thinks it would be a good idea to sort the bubbles into buckets to store for the winter.

Each bubble must be placed in a bucket and she would like the bubbles in each bucket to be as close to the same size as possible. The score of a bucket is the square of the difference in size of the largest and smallest bubble in that bucket. Belinda may put as many or as few bubbles into each bucket as she wishes. If a bucket is empty, its score is 0.

For example, if the buckets contained:  $[1, 2, 3]$ ,  $[3, 5]$ ,  $[4, 4]$ , the total score would be  $(3 - 1)^2 + (5 - 3)^2 + (4 - 4)^2 = 8$ . Given the sizes of the bubbles, what is the minimum total score that Belinda can achieve?



### Input

The first line contains two integers  $n$  ( $1 \leq n \leq 200$ ), which is the number of bubbles, and  $b$  ( $1 \leq b \leq 200$ ), which is the number of buckets.

The next  $n$  lines describe the bubbles. Each of these lines contains a single integer  $k$  ( $1 \leq k \leq 10^9$ ), which is the size of this bubble.

### Output

Display the minimum total score that Belinda can achieve.

#### Sample Input 1

```
5 3
1
2
3
4
5
```

#### Sample Output 1

```
2
```

#### Sample Input 2

```
3 2
117
1337
101
```

#### Sample Output 2

```
256
```

This page is intentionally left (almost) blank.

# Problem C

## Cartography

Time limit: 3 seconds

Cartography is the practice of creating maps. Abbey is a cartographer who wishes to draw a map of where every house is in a particular city. Unfortunately, she has never visited that city. She knows that the city is a rectangular grid with one house at each intersection. She does not know the dimensions of the grid, but knows that each dimension is at least 2.

Abbey has access to the census results from the previous year, where each household in that city wrote down who all of their neighbours were. Your neighbours are the houses that are exactly one intersection away from you in any of the four directions (north, east, south, west). This means that each house will have exactly 2, 3 or 4 neighbours (2 if you are a corner, 3 if you are on the edge, and 4 otherwise).

Unfortunately, some of the census results may be incorrect. For example, some households may have reported the wrong person as their neighbour, some may have not reported some neighbours, some may have reported extra neighbours and some may have even put themselves as their own neighbour!

Help Abbey draw a map of the city that is consistent with all of the census results if possible. The city map is consistent if every household wrote down exactly their neighbours.



### Input

The first line contains a single integer  $n$  ( $4 \leq n \leq 200$ ), which is the number of houses in the city.

The next  $n$  lines describe the census results in order from house 1 to house  $n$ . Each of these lines starts with an integer  $k$  ( $2 \leq k \leq 4$ ), which is the number of neighbours of this house. Then follow  $k$  distinct integers between 1 and  $n$  inclusive, which are the house numbers of the neighbours.

### Output

If there is no valid map that is consistent with the input, display  $-1$ .

Otherwise, display a map of the city. The first line of a map is two integers  $r$  and  $c$ , which are dimensions of the map. Then display a grid of  $r \times c$  integers. The numbers 1 to  $n$  should appear exactly once and the map should be consistent with the input.

If there are multiple correct solutions, any one will be accepted.

#### Sample Input 1

```
4
2 2 3
2 1 4
2 1 4
2 2 3
```

#### Sample Output 1

```
2 2
1 3
2 4
```

#### Sample Input 2

```
4
2 1 2
2 1 2
2 1 2
2 1 2
```

#### Sample Output 2

```
-1
```

**Sample Input 3**

```
6
2 2 3
2 1 4
3 1 4 5
3 2 3 6
2 3 6
2 4 5
```

**Sample Output 3**

```
3 2
1 2
3 4
5 6
```

**Sample Input 4**

```
7
2 4 3
3 1 3 6
4 2 3 7 1
3 1 2 6
2 2 3
2 5 4
2 3 6
```

**Sample Output 4**

```
-1
```

## Problem D

### Divisionals Spelling

Time limit: 1 second

Emilio's team is competing at the South Pacific ICPC Divisional Finals. They have read the problems and immediately know all the solutions. Emilio finds simply winning the contest boring. Instead, he has convinced his team to spell out a word with their submissions. He has put together a list of  $n$  words that he would like to spell.

Each question in the contest is labelled with an uppercase letter. The first question is labelled "A", the second is labelled "B", and so on. A word is spelled by solving questions in a specific order. For example, if Emilio's team wants to spell LEAK, the team first solves problem L, then problem E, then problem A, and finally problem K. The team may only solve each problem once, so they may not spell words with duplicated letters (for example, they cannot spell EMILIO). Which words can Emilio's team spell?



### Input

The first line contains two integers  $n$  ( $1 \leq n \leq 100$ ), which is the number of words in Emilio's list, and  $m$  ( $1 \leq m \leq 15$ ), which is the number of questions in the contest.

The next  $n$  lines describe the words. Each of these lines contains a word with at least 1 and at most 15 uppercase letters. All the words are distinct.

### Output

Display the number of words in the list that can be spelled.

#### Sample Input 1

```
6 12
CAD
ADD
ALE
DALE
PAT
ICPC
```

#### Sample Output 1

```
3
```

#### Sample Input 2

```
4 15
KEK
LMAO
LEL
LOL
```

#### Sample Output 2

```
1
```

This page is intentionally left (almost) blank.

## Problem E

### Entering Rectangles

Time limit: 3 seconds

Robin is using a basic paint tool. This paint tool is so basic that it performs only a single operation. One selects four corner pixels on the screen. If these four pixels form an axis-aligned rectangle, then the tool will change the colour of the pixels on the border of that rectangle to black. If any of these pixels are already black, the program will crash.

Robin has a black and white image. They wish to draw more rectangles on this image. What is the maximum number of rectangles they can draw with this tool without the program crashing?



### Input

The first line contains two integers  $r$  ( $1 \leq r \leq 100$ ) and  $c$  ( $1 \leq c \leq 8$ ), which are the number of rows (in pixels) and columns (in pixels) in Robin's image.

Following this is an  $r \times c$  grid of characters. If a pixel is black, its corresponding character is an asterisk (\*). If a pixel is white, its corresponding character is a dot (.) .

### Output

First, display  $n$ , which is the maximum number of rectangles that can be drawn onto the image.

Then display the image after drawing  $n$  rectangles (indexed 1 to  $n$ ) on it using the basic paint tool. Each pixel should be represented by exactly one integer. If the original pixel was black, display  $-1$ . If the original pixel was white and it was not changed by the basic paint tool, display 0. Otherwise, display the index of the rectangle that was drawn onto that pixel.

If there are multiple possible solutions, any one will be accepted.

#### Sample Input 1

```
3 3
...
...
...
```

#### Sample Output 1

```
1
1 1 1
1 1 1
0 0 0
```

#### Sample Input 2

```
3 5
.....
...*.
**...
```

#### Sample Output 2

```
2
1 1 2 2 2
1 1 2 -1 2
-1 -1 2 2 2
```

#### Sample Input 3

```
4 4
....
....
....
....
```

#### Sample Output 3

```
4
1 1 2 2
1 1 2 2
3 3 4 4
3 3 4 4
```

This page is intentionally left (almost) blank.



## Problem F

### Fabulous Photos

Time limit: 6 seconds

Erica has a collection of bowling balls numbered 1 to  $n$ . Each of her bowling balls is a certain colour and its number is written on the ball. Note that multiple bowling balls may be the same colour.

Erica transports her bowling balls in bags which match the colour of the balls they contain. For example, red bowling balls must go in red bags, green bowling balls must go in green bags, and so on.

Each coloured bag is designed to hold exactly some (possibly empty) subset of the balls of that colour. The bag must hold exactly the subset of balls it is designed to hold when it is used. No two bags of the same colour can hold the same subset of bowling balls.

Erica decided to get her bowling ball collection professionally photographed. For each picture, Erica picked exactly one bag of each colour. She then brought those bags and the bowling balls they hold to the photo studio. The photographer then took all the bowling balls Erica brought out of the bags and lined them up for a picture.

Because Erica is really proud of her bowling ball collection, she went to the studio once for every possible valid combination of bags she could bring.

For example, assume Erica has 5 bowling balls in her collection numbered from 1 to 5. Bowling balls 1, 2, and 5 are red while bowling balls 3 and 4 are green. Also assume that Erica has two red bags: the first of which can hold balls 1 and 2 and the second can hold balls 2 and 5. Erica also has three green bags: the first of which can hold balls 3 and 4, the second of which can hold only ball 3, and the third which is green but can hold no bowling balls. Therefore, Erica can pick either of the two red bags and one of the three green bags. This gives the following six combinations that she will have photographs of: [1, 2, 3, 4], [2, 3, 4, 5], [1, 2, 3], [2, 3, 5], [1, 2], and [2, 5].

Sadly, after Erica got the pictures back from the professional photographer, she realized that the pictures were taken in black and white! This means that each picture only shows the numbers on the balls in the picture and not what colour each ball is.

Given Erica's photographs, can you figure out which bowling balls in her collection are definitely the same colour?

For example, assume Erica had two pictures: the first of which shows balls 1 and 2 and the second of which shows balls 1 and 3. There are two possibilities:

- All bowling balls are the same colour with bags holding {1, 2} and {1, 3}.
- Bowling balls 2 and 3 are the same colour but different from ball 1, with bags holding {1}, {2}, and {3}.

In both cases, bowling balls 2 and 3 are the same colour, while bowling ball 1 may be a different colour.

### Input

The first line contains two integers  $n$  ( $1 \leq n \leq 60$ ), which is the number of bowling balls Erica has, and  $m$  ( $1 \leq m \leq 10\,000$ ), which is the number of pictures Erica took.

The next  $m$  lines describe Erica's pictures. Each picture is encoded as a string of  $n$  0s and 1s describing the balls from 1 to  $n$  in order. A 0 means that the ball was not present in the picture and a 1 means that the ball was included in the picture.

### Output

For each bowling ball in Erica's collection, in order, display the smallest numbered bowling ball that is definitely the same colour as this bowling ball.



**Sample Input 1**

```
3 2
110
101
```

**Sample Output 1**

```
1 2 2
```

**Sample Input 2**

```
6 6
110101
110011
100011
011101
011011
001011
```

**Sample Output 2**

```
1 2 1 2 2 6
```

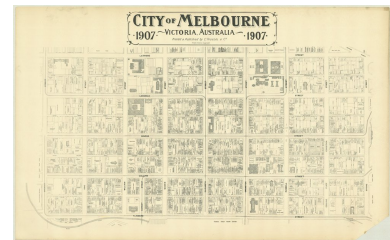
## Problem G

### Grid City

Time limit: 3 seconds

Grid City is growing rapidly. The city council has decided to establish two separate administrative parts: one for downtown and one for the outskirts. They have fixed the boundary, which has the shape of a convex polygon.

A glance at the map reveals that the city is made up of blocks separated by a grid of north-south avenues and east-west streets. A block is the area enclosed between two adjacent streets and two adjacent avenues. Streets and avenues are equally spaced, so all blocks have the same square shape. You can ignore the width of streets and avenues. A block belongs to downtown if it lies entirely within the boundary polygon (the block may touch the boundary). If part of the block (or the whole block) lies outside of the boundary polygon, the entire block belongs to the outskirts.



Source: Melbourne Heritage Action

There is a street running from one of the westernmost vertices of the boundary polygon to one of its easternmost vertices.

Your have to determine the number of blocks in downtown.

### Input

The first line contains an integer  $n$  ( $3 \leq n \leq 100\,000$ ), which is the number of vertices of the boundary polygon.

The next  $n$  lines describe the vertices of the boundary polygon. Each of these lines contains two integers  $x$  ( $1 \leq x \leq 10^9$ ) and  $y$  ( $1 \leq y \leq 10^9$ ). The vertex is on the intersection of avenue  $x$  and street  $y$ .

All vertices are unique and are not all collinear.

### Output

Display the number of blocks in downtown.

#### Sample Input 1

```
4
10 10
50 30
10 30
50 10
```

#### Sample Output 1

```
800
```

#### Sample Input 2

```
5
10 30
20 50
30 10
40 40
50 30
```

#### Sample Output 2

```
805
```

This page is intentionally left (almost) blank.

# Problem H

## Heroic Heist

Time limit: 3 seconds

It is the night before the grand opening of a new art gallery. The gallery consists of  $n$  rooms, numbered from 1 to  $n$ . The rooms are organized sequentially, with room 1 being connected by a door to room 2, and room 2 being connected to room 3, and so on. Each room has a door that leads into it from the preceding room. That door is either locked or unlocked. If the door is unlocked, the room will contain a key. Otherwise, it will not contain a key.



To enter a room with a locked door, you must use a key that is compatible. Each key can open a subset of the doors. The gallery uses a special lock and key system to deter thieves. A key can only be used once, since a locked door will consume any key used to open it.

You start in the first room, which is guaranteed to contain a key, and would like to enter as many rooms as possible. The more rooms you can enter, the more paintings you can... admire.

Assuming you use keys optimally, what is the maximum number of rooms you can enter?

### Input

The first line contains a single integer  $n$  ( $2 \leq n \leq 300$ ), which is the number of rooms.

The next  $n$  lines describe the rooms in the gallery in order. Each of these lines contains either:

- A single integer,  $0 < x < n$ , if that room contains a key. Then,  $x$  integers, listing the numbers of rooms that contain locked doors that the key can open. No room will appear more than once in this list.
- A single integer, 0, if that room has a locked door from the preceding room.

The first room is guaranteed to have  $x > 0$ .

### Output

Display the maximum number of rooms you can enter.

#### Sample Input 1

```
7
3 2 6 7
0
2 2 7
2 5 6
0
0
0
```

#### Sample Output 1

```
5
```

#### Sample Input 2

```
6
3 4 5 6
2 4 5
1 4
0
0
0
```

#### Sample Output 2

```
6
```

This page is intentionally left (almost) blank.

# Problem I

## Iguana Instructions

Time limit: 1 second

Iggy the Iguana has found himself trapped in a corn maze! The corn maze can be modelled as a square grid where some of the cells are blocked off with impassable corn plants and others are cleared out. Iggy can only travel in and through cells that are cleared out. Iggy can move to a cell in any of the four cardinal directions (north, south, east, and west).

Iggy is not good at mazes and needs your help. He has asked you to write down a list of instructions to show him how to reach the end of the maze. Each instruction has the form `<direction> <amount>` where `<direction>` is either North, South, East, or West and `<amount>` is how many cells Iggy should travel in that direction. Because Iggy has a bad memory, he wants this list of instructions to be as short as possible even if that means he has to walk further.

Iggy starts in the top-left cell of the maze and needs to get to the bottom-right cell. It is guaranteed that there exists a path Iggy can take to reach the end.

What is the minimum number of instructions you can give Iggy so that he can reach the end of the maze?



### Input

The first line contains  $n$  ( $2 \leq n \leq 100$ ), which is the length of one side of the square grid representing the maze.

Following this is an  $n \times n$  grid of characters. If a cell is cleared out, its corresponding character is a dot (.). If a cell is blocked off with corn plants, its corresponding character is a hash (#).

### Output

Display the minimum number of instructions you can give Iggy such that he can reach the end of the maze.

#### Sample Input 1

```
5
.....
####.
.....
.####
.....
```

#### Sample Output 1

```
5
```

#### Sample Input 2

```
5
.....
.###.
.....
.####
.....
```

#### Sample Output 2

```
2
```

### Sample Input 3

```
7
.....
#.#.#.#.
#.....#
..####.
#.....##
...#...
##.....
```

### Sample Output 3

```
5
```

### Sample Input 4

```
31
.....
.....
.....
.....
.....##.....
.....##..#.#.#.#.....
.....#.#.#.#.#.#.#.....#####.....
.....#.#.#.#.#.#.#.#.....##.....
..##..##.#####.#.#.#.#.....#..
..#.#.#.#.....#.#.#.#.#..
..#.#.#.....#.#.#.#..#
...#.....##.....#
..#.....##.#
.##.....##.....#####.....##
.#.....#.#.#.#.#.#.#.....#
.#.....#.#.#.#.#.#.#.....#
#.....#.#.#.#.#.#####.....#
#.....#.#.#.#.#.#.#####.....
#.....##.#.#.#.#.#.#.....#
#.....###.....###.#.#.#.....
.##.....##.....#.#.#.....
..##.....##.....#.....
.....###.....#.....
.....##.#.....
.....#.#.....
.....###.....
.....
.....
.....
.....
```

### Sample Output 4

```
11
```



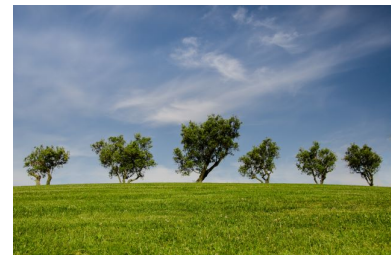
## Problem J

### Jumping Junipers

Time limit: 4 seconds

Jack and Jill have a house in the woods. They have planted some juniper trees in a straight line that goes straight away from their house. Each tree is planted an integer distance from their house along this line. No two trees are at the same location.

Jack and Jill would like to move the trees closer to their home. Since the trees are heavy, they can only move the trees a certain distance in either direction (the distance for each tree can be different). The trees must end up at positive integer distances from their house along the line and no two trees may end up at the same place. Jack and Jill would like to minimize the sum of distances of the trees from their house. Show them how to do this.



### Input

The first line contains a single integer  $n$  ( $1 \leq n \leq 200\,000$ ), which is the number of trees.

The next  $n$  lines describe the trees. Each of these lines contains two integers  $d$  ( $1 \leq d \leq 10^9$ ), which is the distance of this tree from their house, and  $t$  ( $0 \leq t \leq 10^9$ ), which is the maximum distance that this tree can be moved in either direction.

### Output

Display the new locations of the  $n$  trees (in the same order as given in input) that minimizes the sum of distances from Jack and Jill's house.

If there are multiple solutions, any one will be accepted.

#### Sample Input 1

```
3
12 3
11 1
100 1
```

#### Sample Output 1

```
9 10 99
```

#### Sample Input 2

```
2
10 2
12 4
```

#### Sample Output 2

```
8 9
```

#### Sample Input 3

```
2
343 117
100 15
```

#### Sample Output 3

```
226 85
```

This page is intentionally left (almost) blank.

# Problem K

## Karaoke Meetup

Time limit: 8 seconds

The judges of the South Pacific Programming Contest are planning their next secret karaoke meetup and are looking for a place to hold it. Last time, they asked Timothy to pick the location, but of course, he just picked somewhere really close to his house, and far from everyone else's! This year, you would like to pick a meeting location that is fair.

All the judges live in the same city. The city consists of various locations in which the meeting could be held and roads that connect pairs of locations. The city is built such that for each pair of locations, there is exactly one path between them. Each road has a length and can be used to travel in either direction. You consider a meeting point fair if the distances from each of the judges' houses are similar. For each location, its *fairness score* is the ratio  $\frac{A}{B}$ , where  $A$  is the minimum distance from the location to any judges' house and  $B$  is the largest distance. What is the maximum fairness score for all vertices?



### Input

The first line contains two integers  $n$  ( $2 \leq n \leq 200\,000$ ), which is the number of locations in the city, and  $k$  ( $2 \leq k \leq n$ ), which is the number of judges.

The next  $k$  lines describe the location of the judges' houses. Each of these lines contains a single integer  $\ell$  ( $1 \leq \ell \leq n$ ), which is the location of this judge's house. No two judges live at the same location.

The next  $n - 1$  lines describe the roads in the city. Each of these lines contains three integers  $u$  ( $1 \leq u \leq n$ ),  $v$  ( $1 \leq v \leq n$ ), and  $w$  ( $1 \leq w \leq 10^9$ ), which denotes a road between locations  $u$  and  $v$  with a length of  $w$ .

### Output

Display the maximum fairness score as a reduced fraction.

#### Sample Input 1

```
3 2
2
3
1 2 1
1 3 1
```

#### Sample Output 1

```
1 / 1
```

#### Sample Input 2

```
2 2
1
2
1 2 10
```

#### Sample Output 2

```
0 / 1
```

#### Sample Input 3

```
5 3
5
2
4
3 1 1
1 4 1
1 2 1
3 5 1
```

#### Sample Output 3

```
1 / 2
```

**Sample Input 4**

```
10 4
1
5
8
9
3 4 5
3 2 20
4 9 5
2 8 6
6 2 3
5 7 7
10 2 4
4 1 17
7 2 5
```

**Sample Output 4**

```
5/16
```